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Modelling of pulse-periodic energy flow action on metallic materials

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Abstract—Heat processes in pulse-periodic energy flow action on metallic materials are considered. Heating, melting, evaporation and solidification are analysed by means of mathematical modelling. Velocities and positions of phase boundaries (both evaporation and melting) are determined over a wide range of operating parameters. The existence of surface temperature, melt thickness and velocities of phase boundaries of different types of oscillation regimes are shown. Relationships between the pulse-periodic energy flow structure and the evolution of heat processes are determined.

1. INTRODUCTION

THE PULSE and pulse-periodic treatment of metallic materials by means of laser action, electron beam and plasma flows is a well-known and widely used technique [1, 2]. Various types of solid state and CO_2 pulse and pulse-periodic lasers, electron beam guns and plasma torches are available. They are characterized by different energy and pulse durations ($q_0 = 10^3$–10^8 W cm^{-2}, $t = 10^{-9}$–10^{-3} s) and are utilized in a variety of applications including transformation hardening, welding, cutting and alloying. The task of optimizing the operating parameters in pulse-periodic energy flow action is a crucial step. The short duration of pulse action and therefore the high velocities of the heat processes on one side and the localization of action on the other, makes a direct experimental investigation of these phenomena a hard task. Therefore, providing a numerical experiment instead of a real one, is an important opportunity to optimize the parameters of pulse-periodic treatment. On the other side, investigation of the behaviour of phase boundaries during pulse-periodic energy flow action, with different energy distributions, is a fundamental problem.

In a number of articles concerning the problem of the treatment of mathematical modelling of materials by concentrated energy flows, mainly pulsed or continuous conditions of action are considered [3–15] whereas only in some articles are the peculiarities of pulse-periodic action analysed [16–19]. This derives from the difficulties in the simultaneous description of heating/cooling, melting/solidification and evaporation phenomena. Usually only heat transfer during pulse-periodic energy flow action is analysed [16–18] or only the movement of the melting front in one direction (i.e. in the absence of solidification [19]). We are not acquainted with papers where the movement of both phase fronts—evaporation and melting/solidification—are considered concerning the pulse-periodic action of energy flows. Only on the basis of the exact determination of the positions of both phase fronts is it possible to analyse the dependence of the melt thickness on time, which is important for the technological applications of pulse-periodic action of energy flows.

The main aim of the present article is to determine the relationships between the structure (i.e. pulse duration, duty cycle, value of energy density flow, etc.) of energy density flow and the evolution of heat processes.

2. MATHEMATICAL MODEL

The mathematical model proposed includes the processes of heating, melting, evaporation and solidification under the action of an energy flow with different shapes on a metal slab [20–22]. It is assumed that the energy flow is absorbed on the irradiated surface; convection and radiation mechanisms of heat losses from both sides of the slab are considered and melting (solidification) is determined by the classical Stephan boundary condition.

In the present model we neglect the convective heat transfer in comparison with the conduction one. It is necessary to discuss this assumption in detail. In the general case of the action of concentrated energy flows on metals the convective heat transfer is caused mainly by the following reasons: (1) free convection; (2) surface tension driven convection (Marangony effect); (3) forced convection under the action of the evaporation reactive pressure; (4) forced convection under the action of gas or plasma flows. The above-mentioned phenomena are discussed below.

NOMENCLATURE

$a_{1,2}$	thermal diffusivity of liquid and solid phase, respectively
g	free fall acceleration
h	depth of the melt
L	thickness of the slab
L_m	latent heat of melting
L_v	latent heat of evaporation
Pr	Prandtl number
q	energy density flow
$q_0(t)$	absorbed energy density flow
R	radius of the molten pool
Re	Reynolds number
Re^*	reduced Reynolds number, $(v_r k/v)(h/R)^2$
$S_1(t)$, $S_2(t)$	positions of evaporation and melting phase boundaries, respectively
t_m	starting time for melting
T_0	initial temperature
$T_1(x, t)$	temperature of liquid phase
$T_2(x, t)$	temperature of solid phase
T_m	temperature of melting
v_r	radius component of liquid velocity
x, t	distance and time, respectively.

Greek symbols

α	$-\mathrm{d}\sigma/\mathrm{d}T$
$\alpha_{g,r}$	coefficient of convection heat losses of irradiated and rear surfaces of the slab
β	bulk thermal expansion coefficient
$\varepsilon_{1,2}$	emissivities of irradiated and rear surfaces of the slab
$\varepsilon_{g,r}$	emissivities of the environment near the irradiated and rear surfaces of the slab
η	dynamic viscosity
$\lambda_{1,2}$	thermal conductivity of liquid and solid phase, respectively
v	kinetic viscosity
$\rho_{1,2}$	densities of liquid and solid phase, respectively
σ	surface tension.

Free convection

If the treated sample is disposed horizontally and the energy flux acts on its upper surface (typical situation, for example, for laser treatment) the temperature gradient vector and the vector of the free fall acceleration are oriented in opposite directions. This statement is true for the surface heat source, because in most of the cases of the action of concentrated energy flows (laser, plasma, concentrated solar energy, electron beam with comparatively small acceleration voltage) the heat source can be considered flat [1]. In the case of another arrangement of the treated sample the criterion for the neglect of the free convection is the inequality, that is the Rayleigh number is much less than unity, $Ra = (g\beta h^3 \Delta T/va) \ll 1$. For melt thicknesses h of about 50 μm and melt overheating ΔT of about 1000 K the Rayleigh number is of the order of 0.1.

Surface tension driven convection [23–25]

The criterion for the neglect of the convective heat transfer is $Re^*\,Pr = (\alpha q/\eta a\lambda)(h^2/R)^2 \ll 1$. For the typical cases of pulse laser treatment with the duration of nearly 1 ms the corresponding melt thickness is less than 100 μm and the radius of the molten pool is of the order of 1 mm [1]. For a shorter pulse duration, as considered in the present article, the melt thickness is much less but the radius which is determined by the corresponding radius of the laser beam (or another energy flow) is practically the same. Therefore, for the typical conditions of the energy flow action considered here the molten pool is shallow, i.e. $h/R \gg 1$. This leads to consider the conductive heat transfer dominant in comparison with the convective one. As shown previously in ref. [25] the criterion $Re^*\,Pr \ll 1$ is correct for a surface driven convection up to times greater than 1 ms.

Forced convection under the action of the evaporation reactive pressure [1]

The evaporation reactive pressure can be roughly estimated as

$$p \approx A \exp(-T_*/T)$$

where A and T_* are constants. The convective heat transfer will be negligible if

$$Re^*\,Pr \approx (A \exp(-T_*/T)/\eta a)(h^2/R)^2 \ll 1.$$

This inequality usually is correct for a shallow molten pool the surface temperature of which is less than the boiling temperature (this corresponds to values of absorbed energy density flux usually less than 10^6 W cm^{-1} and comparatively short energy pulse duration).

We do not consider the forced convection under the action of gas or plasma flows.

It is possible to conclude that during the action of concentrated energy flows on metals for sufficiently short pulse duration (or limited values of energy density flow) and large radius of energy flux, the convective heat transfer (in a shallow molten pool with the surface temperature less than the corresponding boiling temperature) is less than the conductive heat transfer.

A further question to be discussed is the possibility of using a classical parabolic type equation for the description of high frequency temperature oscillations. In phenomenological heat transfer theory, the velocity of heat propagation is assumed to be infinitely large. In some problems of heat transfer it is necessary to take into account a finite velocity of heat propagation. This idea was first discussed in ref. [26] and later developed in a number of articles [27, 28]. Usually the velocity of heat propagation w is determined by the thermal diffusivity and the relaxation period (time) of heat transfer $w = \sqrt{(a/t_0)}$. The task of heat transfer in a semiinfinite body with constant initial temperature, the surface of which since $t = 0$ is maintained at constant temperature T (the peculiarities of heat transfer in this task seem to be not very far from the problem under consideration here) has been discussed previously [29]. It was shown that starting from the time $t > 8t_0$ the difference between the solutions with finite and with infinite heat transfer velocity is negligible. Similar results can be obtained from other tasks [30]. For different materials and a wider temperature range, the relaxation period is in the range 10^{-10}–10^{-12} s. Therefore, starting from a time larger than 10^{-9} s it is possible to use the standard heat transfer equation of parabolic type.

Close to this question is the problem of photoacoustic pulse generation, which is formulated on the basis of a hyperbolic type of equation [31, 32]. It is assumed that the heat transfer velocity is equal to the longitudinal wave velocity. It is shown that the elastic displacement caused by the surface heat source is of the order of 1 nm and the corresponding typical time values are 150–400 ns. In our article we consider mainly time intervals larger than those mentioned above. Also the existence of liquid phase on the metal surface sufficiently decreases the temperature oscillations in a solid phase (as it will be shown below especially when the amount of heat accumulated in the melt is larger than oscillations of energy flow).

The mathematical model used can be written in the following form:

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{a_1}\frac{\partial T_1}{\partial t} \quad S_1(t) \leqslant x \leqslant S_2(t), \quad t_c \leqslant t < \infty$$

$$q_0(t) - \alpha_g[T_1(x,t) - T_g] - \sigma[c_1 T_1^4(x,t) - c_g T_g^4] = -\lambda_1 \frac{\partial T_1}{\partial x} + \rho_1 L_v \frac{dS_1}{dt}, \quad \lambda = S_1(t)$$

$$\frac{dS_1}{dt} = \frac{v_*}{\sqrt{(T_1(S_1(t),t))}} \exp\left[-\frac{T_*}{T_1(S_1(t),t)}\right]$$

$$\lambda_1 \frac{\partial T_1}{\partial x} = \lambda_2 \frac{\partial T_2}{\partial x} - \rho_2 L_c \frac{dS_2}{dt},$$

$$T_1 = T_2 = T_c, \quad x = S_2(t); \quad S_2(t_c) = 0$$

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{a_2}\frac{\partial T_2}{\partial t} \quad S_2(t) \leqslant x \leqslant L$$

$$-\lambda_2 \frac{\partial T_2}{\partial x} = \alpha_f(T_2 - T_f) + \sigma(c_2 T_2^4 - c_f T_f^4), \quad x = L$$

$$T(x, t = 0) = T_0. \quad (1)$$

Constants v^* and T^* are determined by the Herz–Knudsen law of evaporation

$$v^* = \frac{P_v}{2\rho_1(2\pi k/m)^{1/2}} \exp\left[\frac{L_v}{T_v(k/m)}\right],$$

$$T^* = \frac{L_v}{(k/m)} \quad (2)$$

where Boltzmann's constant $k = 1.38 \times 10^{-23}$ J K^{-1}; m is the atomic mass of the slab material, and T_v the boiling temperature corresponding to the pressure p_v [33, 34]. System (1) is solved numerically, the phase change fronts are tracked continuously and the latent heat release is treated as a moving boundary condition. In both regions of liquid and solid phases the moving uniform grids consist of a fixed number of points. Each grid point moves with different velocity. The Crank–Nicolson technique of various derivatives is used [35–37]. We considered the results of pulse-periodic energy flow action on a steel slab, 1 mm thick, with the following heat transfer constants: $a = 5.5 \times 10^{-6}$ m^2 s^{-1}; $\lambda = 29$ W m^{-1} K^{-1}; $L_m = 2.7 \times 10^5$ J kg^{-1}; $L_v = 7.1 \times 10^6$ J kg^{-1}; $T_m = 1800$ K; $T_v = 2750$ K; $T_0 = 300$ K; $\varepsilon = 0.42$.

In all the cases considered below the slab can be considered as a semi-infinite body.

3. RESULTS AND DISCUSSION

3.1. *Action of rectangular energy pulses*

The time dependence of surface temperature, melt thickness, and velocity of melting (solidification) front are represented on Fig. 1. In this case the pulse-periodic energy flow action was modelled considering constant energy density flow pulses separated by time intervals equal to the pulse duration (duty cycle, that means pulse period to pulse duration ratio equal to 2). The exact value of each pulse duration is 1.82×10^{-5} s, the value of energy density flow is 2.5×10^5 W cm^{-2} and the number of pulses is ten. The end of the last energy pulse being at the time 3.458×10^{-4} s; we set the end of the calculation at 3.64×10^{-4} s; therefore ten periods, each consisting of one energy pulse plus the waiting time, are considered. The irradiated surface temperature starts from the initial value of 300 K, which corresponds to a non-dimensional value of $T_0/T_m = 0.162$, and approaches, after the tenth pulse, a value of $1.5T_m$. According to the structure of pulse-periodic action, the dependence of surface temperature on time shows ten local maxima, at the end of the energy pulses. Each of these is higher than the previous one, but the situation for the corresponding minimum is different. The regularity of surface temperature oscillations is broken when the decreasing temperature curves cross the melting point. Starting from the fourth and up to the seventh pulse on the temperature curve, appear the regions of constant melting temperature value, so-called 'steps'. They are the result of melt solidification when first its overheating above the melting point practically disappears and then solidification starts. Practically all the melt solidifies during this stage. The largest values of melt thickness start from near 1 μm for the fourth pulse up to 5 μm for the sixth one. After that, the amplitude of melt thickness oscillations decreases strongly with time. This is the result of heat accumulation in the melt by its overheating. The larger the melt thickness and its overheating, the larger the amount of energy stored in the melt and therefore the smaller the influence of pulse-periodic structure of energy flow on the melting front movement and the smaller the amplitude of melt thickness oscillations. When the melt thickness is small its maximum value per period of energy flow action corresponds to the end of the energy pulse and to the maximum value of surface temperature too. When the melt thickness increases, it takes more time for heat to reach the phase boundary of melting, in this way oscillations of melt thickness will be retarded if compared with the oscillations of energy flow and surface temperature. At the same time on increasing the melt thickness, the melting velocity quickly decreases (see seventh and eighth pulse) due to the melt overheating. Concerning solidification velocity the situation is the following: the increase of solidification rate with time, after the end of the energy pulse, results from a decrease in the melt overheating rate. Then, when the melt temperature reaches prac-

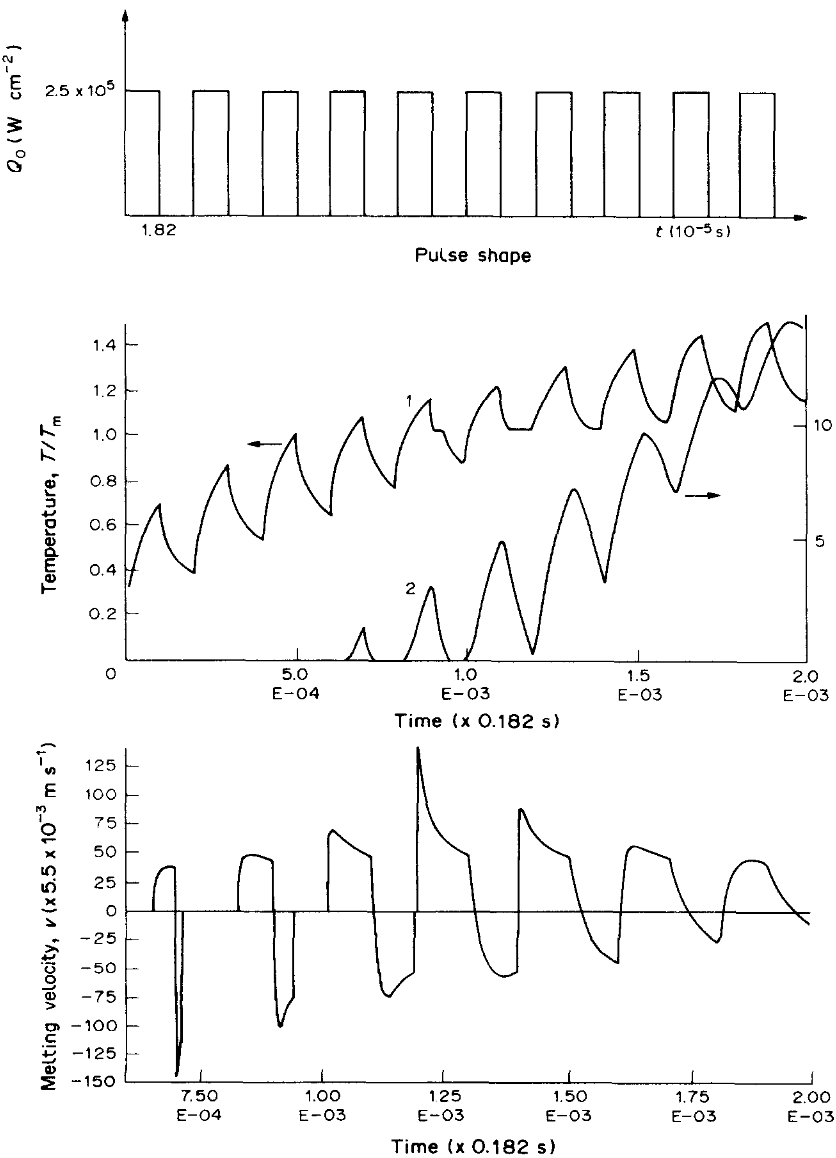

Fig. 1. Heat processes corresponding to the pulse-periodic energy flow action on a steel slab. Shape of energy pulse, rectangular; pulse duration, 1.82×10^{-5} s; duty cycle, 2; maximum value of energy density flow, 2.5×10^5 W cm^{-2}. Time dependence of: density of energy flow (curve 1); surface temperature normalized on the temperature of melting point T_m (curve 2); melt thickness (curve 3); melting (solidification) front velocity (curve 4).

tically the melting point, the solidification rate will be determined by the amount of heat produced at the phase boundary during solidification. In this case the absolute value of the solidification front velocity will decrease and this is the reason why an extreme appears on the curve of solidification velocity. The largest values of solidification velocity correspond to the case of the smallest overheating of the melt, that is the case of energy action on the metal surface corresponding to the smallest melt thickness. This is the reason why the largest value of solidification velocity is in the case of the fourth energy pulse, when the melt just appears. It is necessary to mention the following: the initial value of melting velocity is zero, $v_m(t_m) = 0$, in any case; so that all the melting curves start from the $v = 0$ axis; on the contrary the values of solidification velocity, corresponding to the end of solidification, usually are not equal to zero and the curves do not reach the $v = 0$ axis. For a better understanding of the details of the behaviour of the phase boundary and the exact determination of the values of time, corresponding to the end of solidification, we connected the last points on the solidification curves at the end of solidification with the $v = 0$ axis by straight lines (for the case of the fourth–sixth pulses). When the melt is still present the curves of melting and solidification velocity are continuous and not interrupted. In the considered cases, evaporation does not play an important role because the temperature of the irradiated surface is comparatively small. As an example, the largest values of evaporation velocity corresponding to the eighth, ninth and tenth pulses are respectively 0.045, 0.11 and 0.22 cm s^{-1}, that is less than 1% of the average melting velocity. Practically the same regularities appear when the same energy pulses have been separated by time intervals twice as small (duty cycle = 1.5). In this case melting starts from the third pulse. After the fifth pulse melt does not disappear. After the seventh pulse, melt thickness oscillations are not essential. The maximum absolute values of solidification velocity monotonically decrease with the number of pulses and negative values of phase front velocity practically disappear after the ninth pulse. The evaporation is still weak and the values of the velocities are nearly two times larger than in the previous case.

Considering the same structure of pulse-periodic energy flow as on Fig. 1, but using energy density flow two times larger for each pulse, i.e. $q_0 = 5\times10^5$ W cm^{-2}, the results now show some differences in comparison with the previous ones. Starting from the sixth pulse, surface temperature becomes nearly a periodic function of the time. Beginning from the same pulse (the sixth one) melt thickness becomes practically a non-decreasing function of time with a weakly periodic character, the reason being that melting velocity, starting from the fifth pulse, presents only short periods of negative values, which are one order of magnitude less than the corresponding values of melting velocity. After the eighth pulse, negative values of velocity are disappearing and the phase boundary moves only in one direction. This is the result of heat accumulation in the melt, the thickness of which reaches the value of 28 μm up to the end of the tenth pulse. With the increase of surface temperature values, the evaporation front velocity increases too and becomes comparable to the values of the melting front velocity.

The results of modelling of heat processes under the action of ten rectangular energy pulses ($q = 10^6$ W cm^{-2}) with the same duration as on Fig. 1 and with the duty cycle of 1.5 are considered on Fig. 2. The shape of surface temperature oscillations is almost the same as that of the energy flux. If the maximum values of temperature during the oscillation period starting from the fourth pulse are practically constant, the corresponding minimum values will monotonically increase during the action of seven pulses and only then become constant. This leads to the decreasing amplitude of surface temperature oscillations, the maximum values of which are limited by the evaporation phenomenon and minimum values by heat accumulation near the irradiated surface. It seems that during the action of the last three pulses it is possible to speak of quasi-stationary surface temperature oscillations, which are independent of pulse number.

The behaviour of melt thickness and melting front velocity are even qualitatively different in cases corresponding to Figs. 1 and 2. The damping of melting front oscillations is characterized by a simultaneous monotonic decrease of both maximum and minimum values. The evaporation front velocity values first increase from pulse to pulse, then after some period of time practically copy the profile surface temperature oscillations. When the velocities of both phase boundaries are of the same order of magnitude (last five pulses) then the melt thickness behaviour is determined by both of them. If the decrease of melt thickness after the end of the first pulse is the result of interrupting of surface heating, the decrease of heat thickness on the final stage of energy flow action will be the result of well developed evaporation. In fact, in corresponding time intervals the velocity of the evaporation front is larger than that of the melting front. At the basis of the monotonical increase of average melt thickness values lies: (a) the decrease of its amplitude of oscillations during the first four pulses due to heat accumulation near the irradiated surface; (b) the increase of its amplitude of oscillations during the final pulses as a result of well-developed evaporation. The above-mentioned regularity is more evident for lower energy density flux, when before the stage of well-developed evaporation (appearing consequently later), melt thickness oscillations practically disappear.

3.2. *Action of energy pulses with parabolic shape*

The time dependencies of surface temperature, melt thickness, velocities of melting (solidification) front and front of evaporation are represented in Fig. 3 for

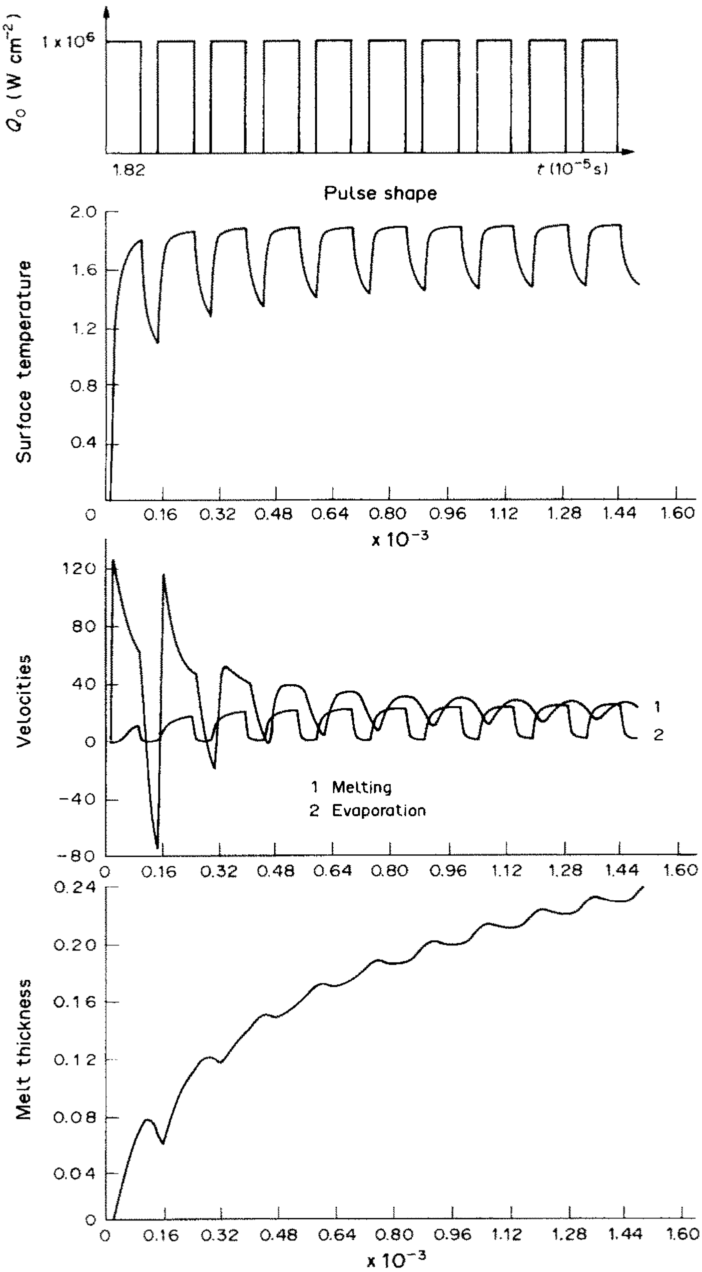

FIG. 2. Heat processes corresponding to the pulse-periodic energy flow action on a steel slab. Shape of energy pulse, rectangular; pulse duration, 1.82×10^{-5} s; duty cycle, 1.5; maximum value of energy density flow, 10^6 W cm^{-2}. Time dependence of: density of energy flow (curve 1); surface temperature normalized on the temperature of melting point T_m (curve 2); melting (solidification) front velocity (curve 3); melt thickness (curve 4).

the case of pulse-periodic energy flow action, where during each period of oscillations with duration of 1.82×10^{-5} s, energy density flow has been represented by a parabolic function with a zero value in the middle of the period and maximum values equal to 5.25×10^5 W cm^{-2} on the boundaries. In this case 20 energy pulses have been considered during the period of energy flow action corresponding to the time of 3.458×10^{-4} s. The calculations were produced up to the time equal to 3.64×10^{-4} s, so that after the end of energy flow action one more time interval, equal to the period of energy flow oscillations can be considered. A mention should be made of the fact that energy input for the first and the last (twentieth) pulse is two times smaller than for all the others.

The irradiated surface temperature profile is characterized by approximately oscillatory behaviour corresponding to the structure of the energy density flow,

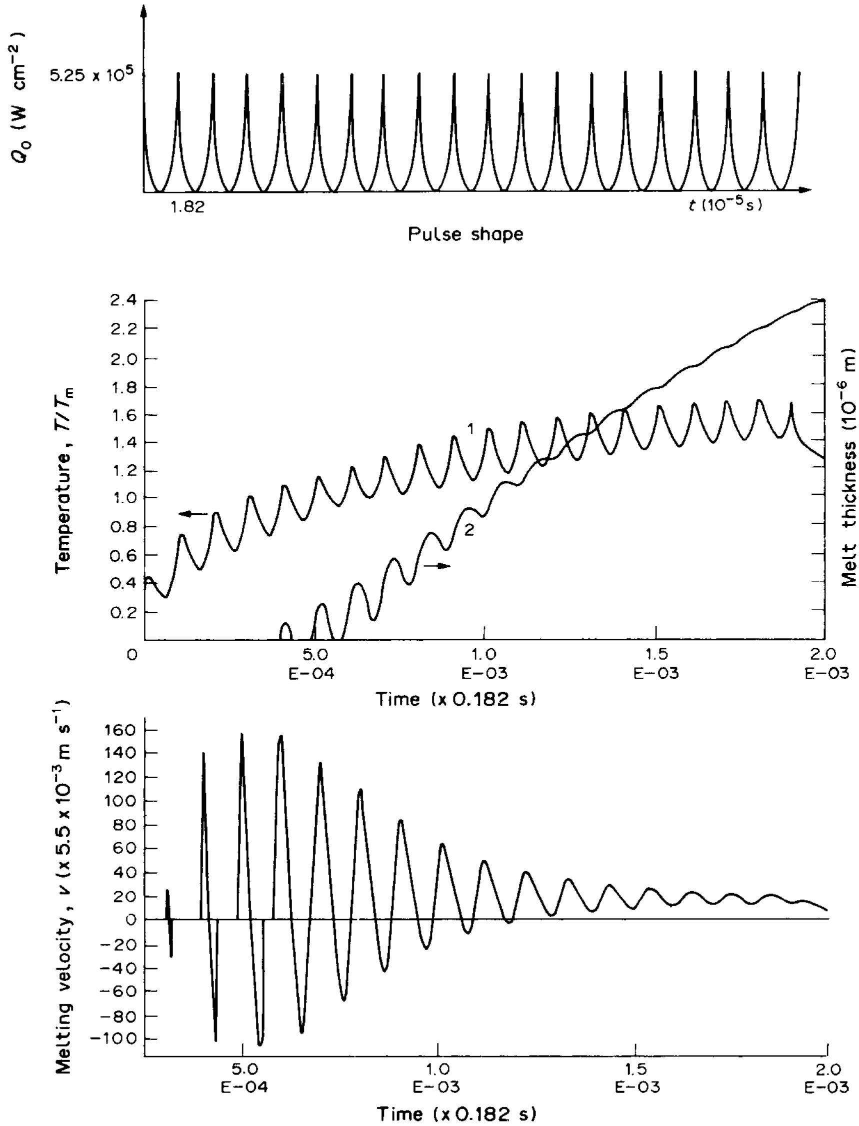

FIG. 3. Heat processes corresponding to the pulse-periodic energy flow action on a steel slab. Shape of energy pulse, parabolic; pulse duration, 1.82×10^{-5} s; duty cycle, 2; maximum value of energy density flow, 5.25×10^5 W cm^{-2}. Time dependence of: density of energy flow (curve 1); surface temperature normalized on the temperature of melting point T_m (curve 2); melt thickness (curve 3); melting (solidification) front velocity (curve 4).

with an exception for the cases of crossing of melting temperature value by the temperature curve. The dependence of melt thickness on time is characterized by a different behaviour at the beginning of the energy flow action (for the case of the first ten pulses) and at the end of action (last ten pulses). The strong oscillatory character is converted in a practically linear dependence. During the action of the fourth, fifth and sixth pulses melt appears and then completely disappears. The dependence of melt thickness vs time of the action of the last ten pulses can be determined by the average energy input and does not depend on the structure of pulse-periodic energy flow. The value of the melt thickness, starting from which the influence of the pulse-periodic structure of energy flow is not significant, is nearly 15 μm, approximately the same as in the previous cases. The most interesting and unusual behaviour demonstrates the dependence of melting (solidification) velocity vs time. The transition from the essential melt thickness oscillations, determined by the energy flow structure to the practically monotonic melt thickness growth determined by the average energy input, is a function of the ratio of the amount of energy stored in the liquid phase to the value of energy flow oscillations relative to its own average. When this ratio is much greater than unity and in the absence of well-developed evaporation, the melt thickness oscillations are weak. The smaller the deviations of pulse-periodic energy flow from its average value, the shorter is the stage of essential melt thickness oscillations. Both deviations of amplitude of energy density flow from its average value and energy input absolute value per oscillation period are significant. On increasing frequency oscillations with unchanged energy flow structure and average energy input value, the melt thickness oscillations will be decreased. The oscillatory type of behaviour of phase boundary velocity, with an initial increase of amplitude oscillations, followed by a further decrease and the weak dependence of oscillations frequency on time seems to be produced rather from a hyperbolic type equation than a parabolic one. Consideration of Fig. 1 reveals the qualitative difference in behaviour of melting (solidification) velocity. The main reason for that is the difference in the pulse-periodic action structure. In Fig. 1 the time intervals of energy action have been followed by the same intervals with the absence of energy flow. In Fig. 3 in one spot per period of action, the value of energy density flow is equal to zero; in all the other time intervals energy flow is small in comparison with the maximum value but it exists. Even a small energy action on a solidifying melt will be significant when the melt thickness is of the order of 1 μm or less, as in the case of fourth–sixth pulses action. Even a light heating of the solidifying melt decreases the absolute value of solidification velocity. The influence of the same energy amount action will decrease with increasing the melt thickness and decreasing the absolute values of solidification velocity. The further action of energy pulses on the metal surface produce the melt thickness and melt overheating enough to stop the solidification phenomenon. This is one more reason for a further decrease in the amplitude of melting velocity oscillations. The average energy input and surface temperature are not large enough for the extensive evaporation phenomenon, the largest velocity values are less than 2 cm s^{-1}.

Considering the same structure of pulse-periodic energy flow as in Fig. 3 but with a double energy density flow, it is possible to see that the oscillations of melting (solidification) velocity have been shifted to the right if compared to the plot of Fig. 3. In this case the increasing absolute values of solidification velocity, with the number of pulses, disappear and the largest absolute values are obtained during early melting stages. The amplitude of melt velocity oscillations decreases down to the value of about 2.5 cm s^{-1}, but on increasing the surface temperature maximum values up to $1.9T_m$, the velocity of evaporation increases too and the maximum value (11 cm s^{-1}) becomes higher than the corresponding values of melting velocity (i.e. about 8 cm s^{-1}).

3.3. *Quasi-stationary state*

Another relevant question for the pulse-periodic energy flow action problem is the existence of a quasi-stationary state. It is well known that in the case of a one-dimensional heating model constant energy density flux causes an unlimited temperature growth, the same is true for the one-dimensional melting model. Only including the evaporation phenomenon, for example in the form of equations (1) and (2), does it become possible to obtain the quasi-stationary solution. Considering the average value of energy density flow instead of the real pulse-periodic structure allows the 'average' time independent solution to be obtained. The details of surface temperature, velocities of phase boundaries and melt thickness changing up to values corresponding to the quasi-stationary state are of great interest. The time dependencies of surface temperature, melt thickness, velocities of melting and evaporation fronts are represented in Fig. 4 for the case of pulse-periodic energy flow action where during each period of oscillations of the same duration as for Figs. 1–3, energy density flow consists of triangle pulses with a zero value at the beginning and the highest value, corresponding to 5.25×10^6 W cm^{-2} at the end of the period. In this case the action of 19 triangle pulses is considered. Practically during the action of the first pulse, the surface temperature reaches its maximum value of about $2.2T_m$. During the further energy flow action the surface temperature copies the structure of energy flow. The evaporation front velocity, defined by surface temperature, basically shows the same behaviour. The largest transient period is in this case for melting front velocity and accordingly to the melt thickness. Only after the action of the sixth pulse does melt thickness reach its maximum value. The value of melt thickness is not constant as in the case of the quasi-stationary solution for constant energy flux but it, like all the other parameters, oscillates with time. In the previous cases the oscillations of melt thicknesses were the result of heating and melting the metal under the action of energy pulses whereas cooling and solidification mainly depend on heat transfer into the bulk of the metal. In Fig. 4 the oscillations of melt thickness are mainly the result of the interaction of melting and evaporation phase boundaries. Both velocities of melting and evaporation have the same average value, both are oscillating, but at the beginning of each energy flow oscillations period the values of melting velocity are larger than the corresponding ones of the evaporation velocity. An opposite situation exists at the end of each period because surface temperature and accordingly evaporation front velocity copies the energy flow structure. This leads to the increase of melt thickness at the beginning and to the decrease at the end of energy pulses. The peculiarity of the considered situation relative to the previous one represented in Figs. 1–3 is in the exact coincidence of the frequency of oscillations of surface temperature, velocities of phase boundaries and melt thickness with the frequency of energy flow. Only in the case when all the physical variables of system (1) will oscillate at one frequency is it possible to obtain their average time independent values for each period of oscillations. Therefore, in

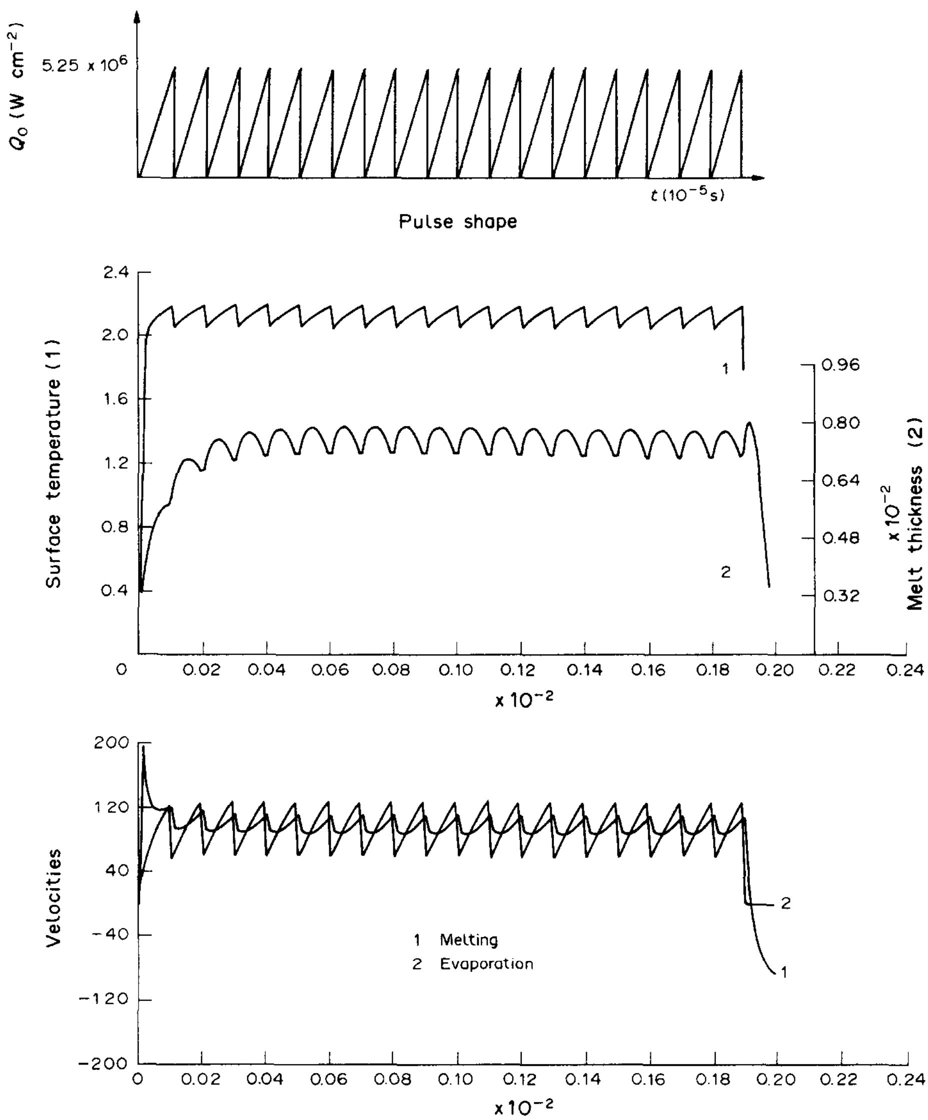

FIG. 4. Heat processes corresponding to the pulse-periodic energy flow action on a steel slab. Shape of energy pulse, triangle; pulse duration, 1.82×10^{-5} s; duty cycle, 2; maximum value of energy density flow, 5.25×10^{6} W cm^{-2}. Time dependence of: density of energy flow (curve 1); surface temperature normalized on the temperature of melting point T_m (curve 2); melt thickness (curve 3); melting (solidification) front velocity (curve 4).

the considered case no exact average time independent values of physical variables exist. After the consideration of the results of pulse-periodic energy flow action with different pulse shape and values of energy density flow is it possible to make the conclusion that the smallest transient period corresponds to the change of surface temperature, the largest to the melt thickness (or velocity of melting) and between them but close to the transient period of surface temperature is the value of the evaporation velocity transient period. For example, consideration of the energy flow with the same triangle structure but two times smaller density of energy flow shows that 19 pulses are not enough for melt thickness stabilization, the average values of which per oscillation period are still growing; instead oscillations of surface temperature and evaporation velocity starting from the sixth pulse are practically independent of pulse number (constant maximum temperature values per period and very weak increase of minimum values). Of course it is possible to reach the quasi-stationary state for average energy density flow two times smaller than in Fig. 4 and for 19 triangle pulses, for example increasing the pulse duration by two and accordingly the time of consideration. In this case the quasi-stationary state is reached just at the end of the pulse series.

4. CONCLUSIONS

The results of mathematical modelling of the action of pulse-periodic energy flow on a metal surface show the necessity of considering the movement of two phase boundaries: melting (solidification) and evaporation fronts. The numerical analysis described shows the possibility to determine a number of regularities in heat processes of the pulse-periodic energy flow action. Surface temperature oscillations follow the energy flow structure except for the cases of crossing the melting point by the temperature curve, where the shape of temperature oscillations may be broken by the energy absorption and release, determined consequently by melting and solidification of metal. As their frequencies are equal to those of energy flows then the amplitude decreases with the increase of energy density flow, time of action and sometimes in the area of crossing the melting point. The latter is characteristic for the relatively low values of energy density flux. The melt thickness behaviour shows a strongly oscillatory character during the initial stage of its existence. Then the amplitude of oscillations strongly decreases up to a practically monotonic increase of melt thickness determined by the average energy input value. With a further increase of energy action time, which leads to the surface temperature increasing and surface evaporation development, melt thickness oscillations appear once again. When the melt thickness is small the frequency of its oscillations is equal to the energy flow one, and then the maximum values of surface temperature and melt thickness per oscillation period are reached at the same time. With increasing melt thickness, the frequency shift appears up to the oscillation of surface temperature and melt depth being in the antiphase. The melting (solidification) velocity is characterized by essentially different types of strongly oscillatory behaviour. During the initial stage of melt existence its amplitude of melting (solidification) velocity oscillations can increase or decrease being determined by the energy flow structure. With a further melt thickness increase, the amplitude of velocity oscillations strongly decreases and the above-mentioned frequency shift appears. At the stage of well-developed evaporation its velocity oscillations are determined by the surface temperature copying its oscillations shape. The existence of quasi-stationary oscillations of surface temperature, melt thickness and velocities of phase boundaries with one frequency (equal to the energy flow one) with the corresponding average values per oscillation period being constant is shown. It is necessary to take into account three different transient periods for the shape of quasi-stationary oscillations: the smallest for surface temperature, the largest for melting front velocities and melt thickness, finally between them for the evaporation front velocity.

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MODELISATION DE L'ACTION D'UN FLUX D'ENERGIE A IMPULSION PERIODIQUE SUR DES MATERIAUX METALLIQUES

Résumé—On considère des mécanismes thermiques sous l'action d'un flux d'énergie à impulsion périodique sur des matériaux métalliques. Le chauffage, la fusion, l'évaporation et la solidification sont analysés par une modélisation mathématique. Les vitesses et les positions des frontières des phases (à la fois évaporation et fusion) sont déterminées pour un large domaine des paramètres opératoires. On montre l'existence de la température superficielle, de l'épaisseur fondue et des vitesses des frontières des phases, pour différents types de régimes. On détermine les relations entre la structure du flux énergétique périodique et l'évolution des mécanismes thermiques.

MODELLIERUNG EINER PERIODISCH GEPULSTEN ENERGIESTRÖMUNG AUF METALLISCHE MATERIALIEN

Zusammenfassung—Es werden Wärmeprozesse mit periodisch gepulster Energieströmung auf metallische Materialien betrachtet. Mit Hilfe eines mathematischen Modells werden die Vorgänge der Beheizung, des Schmelzens, des Verdampfens und des Erstarrens untersucht. Geschwindigkeit und Lage der Phasengrenzen (beim Verdampfen und beim Schmelzen) werden in einem weiten Bereich der Einflußgrößen bestimmt. Für verschiedene Typen von Oszillationen werden Oberflächentemperaturen, Schmelzdicken und Geschwindigkeiten der Phasengrenzen gezeigt. Der Zusammenhang zwischen der Struktur des periodisch gepulsten Energiestroms und der Wärmeausbreitung wird bestimmt.

МОДЕЛИРОВАНИЕ ВОЗДЕЙСТВИЯ ИМПУЛЬСНО-ПЕРИОДИЧЕСКОГО ПОТОКА ЭНЕРГИИ НА МЕТАЛЛИЧЕСКИЕ МАТЕРИАЛЫ

Аннотация—Исследуются тепловые процессы, происходящие при воздействии импульсно-периодического потока энергии на металлические материалы. С помощью математического моделирования анализируются процессы нагрева, плавления, испарения и затвердевания. Определяются скорость и расположение фазовых границ (испарения и плавления) для широкого диапазона изменений режимных параметров. Приводятся данные для температуры поверхности, толщины расплава и скорости фазовых границ при различных типах колебательных режимов. Устанавливается зависимость между структурой импульсно-периодического потока энергии и эволюцией тепловых процессов.